Effect of cavity photons on the generation of multi-particle entanglement

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A study on the cause of the multi-particle entanglement is presented in this work. We investigate how dot-like single quantum well excitons, which are independently coupled through a single microcavity mode, evolve into maximally entangled state as a series of conditional measurements are taken on the cavity field state. We first show how cavity photon affects the entanglement purity of the double-exciton Bell state and the triple-exciton W state. Generalization to multi-excitons W states is then derived analytically. Our results pave the way for studying the crucial cause of multi-particle collective effect.

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The regulation methods of quantum information processing[1] rely on sharing maximally entangled pairs between distant parties. As it is well known, the entangled pairs may become undesired mixed states due to inevitable interactions with environments[2]. For this reason, great attentions have been focused on the agreement of entanglement purification[3], schemes of entanglement distillation[4], and the decoherence mechanisms of quantum bits (qubits) in a reservoir[5].

The environment may play an active role on the formation of the nonlocal effect under well considerations. Many investigations[6] have been devoted to the considerations of the reservoir-induced entanglement between two remote qubits. By manipulating a third system which interacts with two remote qubits[7, 8, 9, 10], many schemes have been proposed to enhance the entanglement fidelity. However, more general situations and considerations are still lacking in a entanglement generation process, especially the multi-particle entanglement generation. This issue is crucial for both the real application of quantum communication and the study on the mechanism of multi-particle entanglement.

In this paper, we study the mechanism of multi-particle entanglement generation. The feasible physical system undertaken is dot-like single quantum well exciton coupled through a single microcavity mode, which is proposed and depicted in Fig.1. The whole procedure in the experimental realization can be performed by optical initialization, manipulation, and read-out of exciton state. In it, the qubit is coded in the presence of an exciton in a quantum well (QW), namely the exciton state in i-th QW, $|e,h\rangle_i$, is considered as the logical state $|1\rangle_i$, and the vacuum state, $|0,0\rangle_i$, which represents the state with no electron and hole, is coded as the logical state $|0\rangle_i$. To analyze the dynamics of the multi-exciton entanglement, a series of conditional measurement can be taken on the cavity field state by means of the electro-optic

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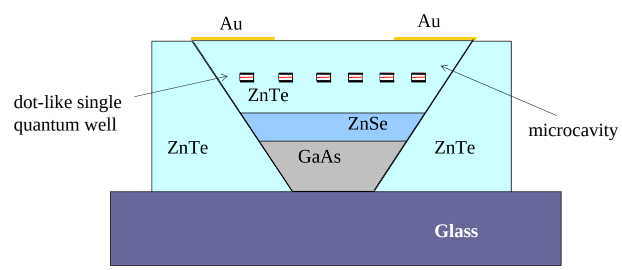

FIG. 1: The quantum devices with six dot-like quantum wells inlaid in a microcavity which is constructed by a ZnTe medium and two Au mirrors. This device can be prepared by the MBE, the e-beam lithography, and the conventional semiconductor processing.

effect. First, we demonstrate how double-exciton Bell state and the triple-exciton W state can be generated under the effect of conditional measurements. Then we discuss the cause of multi-exciton W state, and propose a general formulation of entanglement generation further. Several essentials of effects of the cavity photon will also be presented.

In the QW-cavity system, we assume that the lateral size of the QWs are sufficiently larger than the Bohr radius of excitons but smaller than the wavelength of the photon fields. The dipole-dipole interactions and other nonlinear interactions therefore can be neglected. The cavity mode is assumed to be resonant with the excitons and equally interact with each QW. Under the rotating wave approximation, the unitary time evolution of the n-QWs and cavity field is then governed by the interaction picture Hamiltonian

$$H_{n(I)} = \sum_{m=1}^{n} \hbar\gamma(a\sigma_m^+ + a^+\sigma_m^-), \tag{1}$$

where γ is the coupling constant, a^+ (a) is the creation (annihilation) operator of the cavity field, and σ_m^+ (σ_m) represents the creation (annihilation) operator of the excitons in the mth QW.

For $n = 2$ case and let the whole system be in the m-quanta state, there would be an eigenstate of the Hamiltonian associated with the photon trap, namely, $|\phi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1 |0\rangle_2 - |0\rangle_1 |1\rangle_2) \otimes |m-1\rangle_c$, where $|m-1\rangle_c$ refers to the cavity field state with $m-1$ quanta. Once the system is in this state, the whole system does not decay at all. Accordingly, keeping the cavity mode in state $|m-1\rangle_c$ paves the way to generate the entangled excitons, $|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1 |0\rangle_2 - |0\rangle_1 |1\rangle_2)$. For entanglement generation, the double dot-like single QWs and cavity mode is prepared in the initial state, $|\psi_0\rangle = |1\rangle_1 |0\rangle_2 |0\rangle_c$. For the sake of generality and purpose of distillation, the state of QW can be any mixed state, ρ_ψ, except the vacuum state. Next, a pulse with m photons is injected into the microcavity. For the feasibility and the modest technology requirements, we consider here the injection of single photon, i.e. $m = 1$. The total number of quantum count of the system is two. As the single-photon has been injected into the cavity, the total system will evolve with time, and the evolution operator $U(t)$ can be easily derived form Eq. (1). If the system evolves without interruption, it will go into a QW1-QW2-cavity field entangled state. If we take a measurement on the cavity field state at some instant, the number of the photon count of the detector may be one, two, or zero. Since the single-photon state $|1\rangle_c$ involves the photon-trapping phenomenon, we can infer that the double-QW will evolve into a maximal entangled state if the cavity mode stays in state $|1\rangle_c$ via the quantum jump approach[11]. After measuring the cavity field state, injecting a subsequent photon is necessary for the sake of keeping the photon in its state. We then let the whole system evolve for another period of time τ . Again, we proceed to measure the cavity photon to make sure whether it is one or not. If the cavity photon remains in single-photon, the repetition continues; if not, the whole procedure should be started over.

Therefore, after several times of successful repetitions, the state of double-exciton progresses into the state, $\rho_{\psi_N} = ({}_c\langle 1| U(\tau) |1\rangle_c)^N \rho_{\psi_i} ({}_c\langle 1| U(-\tau) |1\rangle_c)^N / P_{N,n=2}$, where $P_{N,n=2} = \mathrm{Tr}[({}_c\langle 1| U(\tau) |1\rangle_c)^N \rho_{\psi_i} ({}_c\langle 1| U(-\tau) |1\rangle_c)^N]$, is the probability of success for measuring a single-photon after N times of repetitions. The superoperator, ${}_c\langle 1| U(\tau) |1\rangle_c$, reveals the significant fact that it will evolve to the projection operator, $|\psi\rangle\langle\psi|$, as the successful repetitions increases. This result comes from the fact that the superoperator, ${}_c\langle 1| U(\tau) |1\rangle_c$, has only one eigenvalue whose absolute value equals to one[7, 9] and the corresponding eigenvector is just the photon-trapping state. Thus the double-exciton state will become a maximal entangled state after sufficient large repetitions. The probability of success and fidelity can be evaluated as: $P_{N,2} = \frac{1}{2}(1 + \cos(\sqrt{6}\gamma\tau)^{2N})$ and $F_{N,2} = \langle\psi| \rho_{\psi_N} |\psi\rangle = \frac{1}{2\cos(\sqrt{6}\gamma\tau)^{2N}}$. If $\gamma\tau$ is set to be $(2m-1)\pi/(4\sqrt{6})$, $m = 1, 2, ...$, the fidelity of the double-exciton state will approach to one and the probability of success will be 1/2 for large N . If the state of the cavity field is kept in $m-$photon state, rather than the state with single photon, the above results can be generalized to $P_{N,2} = \frac{1}{2}(1 + \cos(\sqrt{2(2m+1)}\gamma\tau)^{2N})$, and $F_{N,2} = \frac{1}{2\cos(\sqrt{2(2m+1)}\gamma\tau)^{2N}}$. It reveals that the number of measured cavity photon and the evolution period play important roles in the trade-off between $P_{N,2}$ and $F_{N,2}$. We can choose a set of $(m, \gamma\tau)$ such that the fidelity progresses to one at the least repetitions, however, in the same time it causes the probability to reduce to a minimum. On the other hand, one can also find a suitability such that the probability goes to one. In this case, the system will not evolve with time and is similar to the Zeno paradox with *finite* duration between two measurements.

We may directly follow the scheme based on continues measurements to achieve the three-particle entanglement generation. However, we have observed that the symmetry of the three-particle Hamiltonian is quite different from the two-particle one, and one may hardly expect what kinds of entangled states can be generated via the quantum jump method and even query whether the quantum jump approach can pave the way for multi-particle entanglement generation. In what follows we will first show that three-particle entangled state indeed can be produced via conditional measurements; moreover the W-type maximally entanglement can rise in multi-QWs system. The superoperator that governs the progress of the three dot-like single QWs can be worked out:

$$\begin{aligned} {}_c\langle 1| e^{iH_{3(I)}\tau} |1\rangle_c &= g\,|g\rangle\langle g| + W_1 |W_1\rangle\langle W_1| + T_1 |T_1\rangle\langle T_1| \\ &+ T_2 |T_2\rangle\langle T_2| + e\,|e\rangle\langle e| + W_2 |W_2\rangle\langle W_2| \\ &+ T_3 |T_3\rangle\langle T_3| + T_4 |T_4\rangle\langle T_4| , \end{aligned} \tag{2}$$

where g, e, W_1, W_2, T_1, T_2, T_3 and T_4 are functions of τ corresponding to the orthonormal eigenvectors

$$\begin{aligned} &|g\rangle = |000\rangle , |W_1\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle , \\ &|T_1\rangle = \frac{1}{\sqrt{2}}(|100\rangle - |001\rangle , \\ &|T_2\rangle = \frac{1}{\sqrt{6}}(|100\rangle - 2\,|101\rangle + |001\rangle , \\ &|e\rangle = |111\rangle , |W_2\rangle = \frac{1}{\sqrt{3}}(|011\rangle + |101\rangle + |110\rangle , \\ &|T_3\rangle = \frac{1}{\sqrt{2}}(|011\rangle - |110\rangle , \text{and} \\ &|T_4\rangle = \frac{1}{\sqrt{6}}(|011\rangle - 2\,|010\rangle + |110\rangle). \end{aligned} \tag{3}$$

Here $T_1 = T_2$ and $T_3 = T_4$ are two two-fold degenerate eigenvalues of the superoperator ${}_c\langle 1| e^{iH_{3(I)}\tau} |1\rangle_c$.

If the initial state is set equal to $|\psi_0\rangle = |100\rangle$, the probability of success for finding the exciton state, $|W_1\rangle$, can then be worked out analytically:

$$P_{N,3} = \frac{1}{3}(\cos(\sqrt{10}\gamma\tau)^{2N} + 2\cos(\gamma\tau)^{2N}), \tag{4}$$

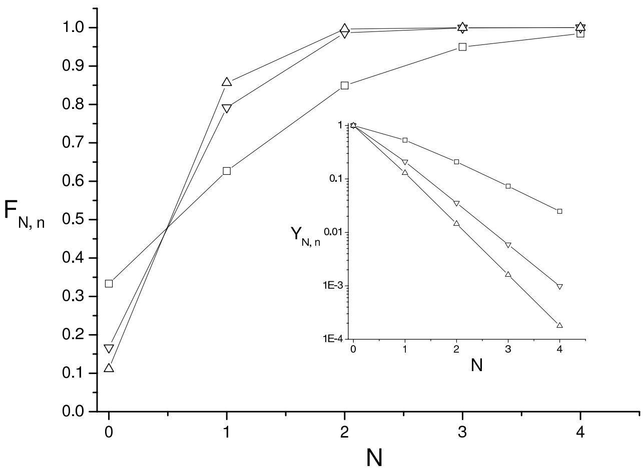

FIG. 2: The variations of fidelity $F_{N,n}$ and the purification yield $Y_{N,n}$(in the inserted diagram) for cases $n = 3(\square), 6(\nabla)$, and $9(\triangle)$, in which the evolution time of each case, $\tau_3 = \pi/(\sqrt{10}\gamma)$, $\tau_6 = \pi/(\sqrt{22}\gamma)$, and $\tau_9 = \pi/(4\sqrt{2}\gamma)$ has been set.

and the corresponding fidelity of the three-exciton state is

$$F_{N,3} = \langle W_1| \rho_{\psi_N} |W_1\rangle = \frac{\cos(\sqrt{10}\gamma\tau)^{2N}}{\cos(\sqrt{10}\gamma\tau)^{2N} + 2\cos(\gamma\tau)^{2N}}. \tag{5}$$

Here, we can set $\gamma\tau$ to be $n\pi/\sqrt{10}$, $n = 1, 2, ...$, then the fidelity of the three-exciton state approaches to unit as N increases; meanwhile the probability of success is 1/3.

If the notion of Plenio et al.[7] is generalized to the case of three particles, in which a leak cavity is used for continuously measuring the vacuum cavity field state $|0\rangle_c$, one can find that the eigenvalue with unit norm of the superoperator, ${}_c\langle 1| e^{iH_{3(I)}\tau} |1\rangle_c$, is three-fold degenerate and the set of corresponding eigenvectors is $\{|000\rangle, |\psi_{ij}\rangle, |\psi_{kl}\rangle\}$, where $|\psi_{ij(kl)}\rangle$ is the Bell-type state in which one photon is trapped and shared between $i(k)$-QW and $j(l)$-QW, $i \neq j$, $k \neq l$ and $i, j, k, l = 1, 2, 3$. Thus the collective motion cannot be induced via monitoring the cavity vacuum.

Now we investigate the general case of multi-QWs entanglement generation further. It is provable that the n-particle one-photon-trapping W state,

$$|\psi_{W1}\rangle = \frac{1}{\sqrt{n}}(|100...0\rangle + |010...0\rangle + ... |0...01\rangle), \tag{6}$$

and the n-particle $(n-1)$-photon-trapping W state,

$$|\psi_{W2}\rangle = \frac{1}{\sqrt{n}}(|011...1\rangle + |101...1\rangle + ... |1...10\rangle), \tag{7}$$

are eigenstates of ${}_c\langle 1| e^{iH_{n(I)}\tau} |1\rangle_c$ by means of the symmetric properties of the system Hamiltonian, operator algebra, and the Taylor expansions of the superoperator ${}_c\langle 1| e^{iH_{n(I)}\tau} |1\rangle_c$, where $n = 3, 4, ...$ However, there exists the only state, $|\psi_{W1}\rangle$, can be generated through conditional measurements, and the selectivity is judged by the criterion of unit norm of which eigenvalues. The eigenvalue of the state $|\psi_{W1}\rangle$ can be derived, namely, $\cos(\sqrt{4n-2}\gamma\tau)$, and its absolute value will be one only at some nodes, viz $\sqrt{4n-2}\gamma\tau = m\pi$, where m is a positive integer. For the case of $n = 6$, we prepare the initial state of six-QWs to be $|100000\rangle$, following the standard procedure of entanglement generation mentioned above and measuring the cavity mode under the condition $\gamma\tau = \pi/\sqrt{22}$, then the six-QWs will evolve to state $|\psi_{W1}\rangle$ with the probability

$$P_{N,6} = \frac{1}{6}(\cos(\sqrt{22}\gamma\tau)^{2N} + 5\cos(2\gamma\tau)^{2N}), \tag{8}$$

meanwhile, with the generation fidelity

$$F_{N,6} = \frac{\cos(\sqrt{22}\gamma\tau)^{2N}}{\cos(\sqrt{22}\gamma\tau)^{2N} + 5\cos(2\gamma\tau)^{2N}}. \tag{9}$$

In which, the time period between two successive measurements can be shorter than the one in the three-exciton case. Besides, the values of $P_{N,6}$ and $F_{N,6}$ approach to steady in fewer steps, but the purification yield will be lower. From the trend of a decrease in success probability, the limitation and efficiency will be an issue when one devises the quantum strategy that can induce a collective motion in the multi-particle system. The generalization of multi-QWs entanglement formulation has been concluded as follows,

$$P_{N,n} = \frac{1}{n}(\cos(\sqrt{4n-2}\gamma\tau)^{2N} + (n-1)\cos(\sqrt{n-2}\gamma\tau)^{2N}), \text{and} \tag{10}$$

$$F_{N,n} = \frac{\cos(\sqrt{4n-2}\gamma\tau)^{2N}}{\cos(\sqrt{4n-2}\gamma\tau)^{2N} + (n-1)\cos(\sqrt{n-2}\gamma\tau)^{2N}}, \tag{11}$$

where $n = 3, 4, ...$ Fig. 2 shows the variations of the probability $P_{N,n}$ and the purification yield $Y_{N,n}$, defined

by $Y_{N,n} = \prod_{i=0}^{N} P_{i,n}$, for the cases of $n = 3, 6$, and 9. It implies that if there is a large dot-like single QW embedded inside the single mode cavity, the collective motion will be hard to exist in the system via conditionally measuring one single photon of the cavity mode. Although as the particle number grows the probability decreases, even at one step the whole particles can be entangled in the W-type state.

Several essentials of multi-QWs entanglement and the collective effect should be expatiated here. First, the W-type entangled states are not the eigenstates of the system, so that the explanation of the progression of entangled QWs based on the quantum jump approach will be quite different from the double-QWs case. Not merely the quantum jump method does elaborate the entanglement formation in decoherence-free subspace, but it specifies the general formulation of multi-particle entanglement. The symmetry of double-QWs-cavity system causes QWs to evolve into a singlet state, hence these states will maintain the coherence through sharing one single photon between them. However, this relation will be broken as the cavity field mode mediates interaction between more than two QWs, consequently another type of entanglement exists in the system, namely the W state. The collective effect, which results in the formation of W states, does not always arise in a multi-particle system. If the conditional result of the measurement depends upon the vacuum of the cavity field and the purification acts like the two-particle scheme of Plenino et al.[7], the QWs will get into a dilemma in which each QW can cooperate with any other QW in the cavity to trap one photon in the same instant due to the zero-quanta cavity mode, that is the superoperator ${}_c\langle 0| e^{iH_{n(I)}\tau} |0\rangle_c$ has a $(n-1)$-fold degenerate eigenvalue, 1, and the pairs of photon-trapping states constitute the corresponding set of orhtonormal eigenvectors. Since the degeneracy inevitably appears in the system under frequently measurements on the vacuum of the cavity field, the notion of Plenino et al. therefore may not be extended to multi-particle entanglement generation. Thus, keeping the cavity mode to retain one photon such that one may block the formation of pair-wise photon trap is the cause of multi-QWs collective effect. Secondly, when the procedure of entanglement generation ends up with a n-particle W state and zero cavity photon, then the QWs and cavity field state will evolve under the dynamics, $\cos(\sqrt{n}\gamma t)\,|\psi_{\mathrm{W1}}\rangle\,|0\rangle_c - i\sin(\sqrt{n}\gamma t)\,|\mathbf{0}\rangle\,|1\rangle_c$, where $|\mathbf{0}\rangle \equiv |0\rangle^{\otimes n}$. If the total number of quanta is two, the evolution of the state, $|\psi_{\mathrm{W1}}\rangle\,|1\rangle_c$, becomes, $\cos(\sqrt{3n-2}\gamma t)\,|\psi_{\mathrm{W1}}\rangle\,|1\rangle_c - i\sin(\sqrt{3n-2}\gamma t)(\sqrt{\frac{n}{3n-2}}\,|\mathbf{0}\rangle\,|2\rangle_c + \sqrt{\frac{2n-2}{3n-2}}\,|\phi\rangle\,|0\rangle_c)$, where $|\phi\rangle \equiv \frac{1}{\sqrt{n(n-1)}}(|110...0\rangle + |1010...0\rangle + ... + |0...011\rangle)$. From which, unlike the decoherence-free state in $n = 2$ case, the generalized W states suffer from Rabi oscillations with the frequencies proportional to the square root of the QW numbers. Finally, the photon number of the cavity field state plays a crucial role of the quantum strategies, based on conditional measurements, that can perform exotic quantum computation. One may accomplish the initialization of the qubits by conditionally monitoring two-photon state of the cavity mode. Since the vacuum state associates with the eigenvector of the superoperator ${}_c\langle 2| e^{iH_{n(I)}\tau} |2\rangle_c$ with eigenvalue $\frac{n(\cos(\sqrt{4n-2}\gamma\tau)+1)-1}{2n-1}$, under a well control of evolution time τ the qubits can then be initialized.

To summarize, a general formulation of multi-QWs entanglement generation based on conditioned measurements has been proposed in this work. We investigate the entanglement generation of dot-like single quantum well excitons coupled through a single microcavity mode. We first consider how the cavity photon affects the purity of the exciton state and the purification efficiency in two- and three-particle protocol. The mechanism of multi-QWs entanglement generation and the cause of collective effect then have been investigated. Thus, we formulate the dynamics of W-type entangled state by the Eqs. (10) and (11). Finally, some essentials of multi-QWs entanglement and the collective effect, including the block of photon trap and the advanced application to quantum computation, have also be studied.

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